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Cosmological Dissipative Structure

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The concept of black hole entropy is generalized to cosmological event horizons. An analogue of the Bekenstein–Hawking generalized second law of thermodynamics is suggested. This law is illustrated by considering entropy changes in various black hole de Sitter spacetimes, and also with the help of a viscous-driven de Sitter universe model, which provides a cosmological version of a far-from-equilibrium dissipative structure. The law apparently fails for some recontracting-universe models. This indicates that a contribution to the gravitational entropy has been omitted. A possible remedy involving algorithmic complexity theory is suggested. I propose the use of a cosmic "entropy censorship" hypothesis as a filter for acceptable field theories.

1. BACKGROUND

One of the most challenging problems of fundamental physics is to explain the origin of time asymmetry in the physical world. There is general agreement (Davies, 1974) that the "arrow of time" must ultimately be rooted in cosmological considerations, possibly by reference to special initial conditions (Penrose, 1979). A proper treatment of time asymmetry in cosmology must take into account the thermodynamics of both the cosmic material and the cosmological gravitational field.

The thermodynamic properties of self-gravitating systems can appear very strange, involving peculiarities such as negative specific heat. Moreover, the theory is replete with difficulties, which have not yet been resolved. The implications of the subject for cosmology have been discussed ever since Tolman's (1934) early work.

In recent years, the thrust of research has been toward attempting to define a gravitational entropy (Penrose, 1979). The stimulus for this direction of investigation comes from Hawking's (1975) famous discovery of black hole radiance, which leads him to associate an entropy with the black hole

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determined by the ratio of its event horizon area with the Planck area. Thus

$$S_{\mathrm{bh}} = A/4 \tag{1.1}$$

where A is the event horizon area and I use units $\hbar = c = G = k = 1$. This leads to a generalized second law of thermodynamics, as first suggested by Bekenstein (1973):

$$\dot{S}_{\mathrm{br}} + \dot{S}_{\mathrm{m}} \geq 0 \tag{1.2}$$

where S_{m} is the entropy of the matter in the environment of the hole, and the overdot indicates differentiation with respect to observer time.

If one regards the black hole as the equilibrium end state of gravitational self-aggregation, then it seems reasonable that (1.1) should be regarded as a form of gravitational entropy, to be added to ordinary matter entropy when discussing the thermodynamics of self-gravitating systems. One can then envisage a further generalization of the second law away from this equilibrium state to more general gravitational fields. In this wider context, the law would encompass the well-known tendency for self-gravitating systems to grow more clumpy with time. Penrose (1979) has discussed how this tendency might relate to the arrow of time in cosmology.

In spite of the intuitive appeal of these ideas, no completely satisfactory mathematical expression has been forthcoming to play the role of gravitational entropy, except in the case of black holes, and the formulation of a consistent theory of thermodynamics for self-gravitating systems remains an outstanding challenge. Special importance attaches to the cosmological case, where the primordial smoothness of the cosmological gravitational field is the key to explaining the cosmological arrow of time (Davies, 1983, 1984*a*). As a contribution to this topic, I here discuss a limited generalization of the Bekenstein–Hawking entropy concept, to spacetimes which possess cosmological event horizons.

2. DE SITTER AND RELATED SPACETIMES

Discussions of why the universe which emerged from the big bang was relatively gravitationally smooth tend to be formulated these days in the context of the inflationary universe scenario [see Turner (1988) for a comprehensive review]. According to this theory, the universe embarked upon a period of de Sitter-like expansion in its primordial phase, during which time the cosmological scale factor $a(t)$ assumed the form

$$a(t) = \exp(Ht) \tag{2.1}$$

where H is a constant, related to the cosmological constant Λ by $H^2 = \Lambda/3$.

For numerical ease I shall define the cosmological constant here as 3Λ, and take $H = \Lambda^{1/2}$.

If the exponential expansion were allowed to proceed for all time, this spacetime would possess an event horizon with area

$$A = 4\pi / H^2 \tag{2.2}$$

(assuming spatial flatness). However, so long as exponential expansion continues for many e-folding times (which it is supposed to do), then there will be an effective de Sitter horizon, even though strictly speaking the notion of event horizon is an asymptotic concept.

Gibbons and Hawking (1977) have argued that de Sitter horizons possess thermodynamic properties closely similar to black hole horizons. In particular, the de Sitter horizon has an associated entropy of $A/4$ as in the black hole case, and a temperature

$$T = H/2\pi = \Lambda^{1/2}/2\pi \tag{2.3}$$

It must be said, however, that the thermodynamic status of de Sitter horizons differs in some crucial respects from the black hole case:

(a) The observer is "inside" the de Sitter horizon, but outside the black hole horizon.

(b) The de Sitter horizon is observer dependent. That is, each observer in de Sitter space will possess a horizon at a proper distance $1/H$, but observers at different positions, or at the same position but moving differently, will locate their horizons differently. In contrast, the black hole horizon is independent of the observer so long as the observer is outside.

(c) Assuming a de Sitter-invariant quantum vacuum state, a static particle detector will respond as though immersed in a bath of thermal radiation at the temperature given by (2.3). However, the stress-energy-momentum tensor of this state is *not* that of thermal radiation. In particular, the pressure p and energy density ρ are not related by $p = \rho/3$. By contrast, the stress-energy-momentum tensor in the spatially asymptotic region of a black hole in the analogous Hartle–Hawking vacuum state *does* correspond to thermal radiation (e.g., Birrell and Davies, 1981).

(d) The total entropy of a black hole in equilibrium with a heat bath consists of two terms: the event horizon area (gravitational entropy) and the conventional entropy of the radiation which constitutes the heat bath. The total entropy of de Sitter space consists of only one term: the event horizon area. If the system is treated as *if* de Sitter space were filled with ordinary thermal radiation at temperature given by (2.3), this radiation would have an entropy equal to that of the horizon area. Including both contributions thus overcounts the entropy by a factor 2.

(e) Unlike in the black hole case, there is no parameter to play the role of mass-energy for de Sitter space. Thus, in discussing the exchange of energy and entropy between de Sitter horizons and matter, the consistent application of the first and second laws of thermodynamics is problematic.

(f) Black hole entropy has a natural pedagogical interpretation in terms of the information lost to the outside universe when the material which formed the hole collapsed gravitationally. In the case of de Sitter space there is no obvious meaning to be attached to the total information content of the matter fields that lie beyond the horizon.

In spite of these differences, there is good circumstantial evidence for a generalized second law of thermodynamics that may be applied to de Sitter horizons. First, it may be shown (Davies, in press-*a*) that there is a limited sense in which de Sitter entropy can be interpreted as "missing information." The technique is to consider an exactly soluble radiation-dominated big-bang Friedmann model universe with a cosmological constant. This model tends to de Sitter space at late time. The event horizon starts out at the big bang with zero area, and grows monotonically toward its constant de Sitter space value. At the same time the radiation flows away across the horizon, leaving the space devoid of matter at late time. It turns out that the initial information content of the matter within a horizon volume is equal to the final horizon entropy.

Second, one may consider (Davies, 1984*b*) a variety of thought-experiments in which boxes of thermal radiation are slowly "lowered" toward a de Sitter horizon, and their contents tipped across the horizon. At first sight this strategy seems to allow for a violation of the ordinary second law of thermodynamics, for the following reason. When the box reaches the horizon, an amount of work equal to its total mass-energy can be extracted by the lowering mechanism. If, then, the contents of the box are tipped over the horizon, no energy exchange takes place. Hence there is no change in the structure of spacetime (in particular, the horizon area remains the same), and no change in the total energy. Yet an amount of entropy equal to the contents of the box has been lost to the region of the universe within the horizon. In analogy to the work of Unruh and Wald (1982) in the black hole case, one finds, however, that complete consistency with the second law of thermodynamics is restored as soon as account is taken of the thermal properties of the de Sitter horizon.

Perhaps the most persuasive evidence in favor of the successful generalization of the second law to de Sitter horizons comes from examining models in which both black hole and de Sitter horizons are present, and then considering a trade of energy and entropy between them. For the purpose of these calculations it is easier to work with the static coordinatization of de Sitter space, rather than with the scale factor (2.1). The metric for a spherical (nonrotating) black hole of mass M and electric charge Q in a de Sitter universe with cosmological constant 3Λ is

$$ds^2 = C(r)\,dt^2 - C(r)^{-1}\,dr^2 + r^2(d\theta^2 + \sin^2\theta\,d\varphi^2) \tag{2.4}$$

where

$$C(r) = 1 - 2M/r + Q^2/r^2 - \Lambda r^2 \tag{2.5}$$

The horizons are located at the roots of the equation $C(r) = 0$, or of the quartic

$$\Lambda r^4 - r^2 + 2Mr - Q^2 = 0 \tag{2.6}$$

the largest root, r_1, being the radius of the cosmological horizon, and the next largest root, r_2, the radius of the black hole horizon. The total horizon area of interest is therefore

$$A = 4\pi(r_1^2 + r_2^2) \tag{2.7}$$

The temperature of a horizon is given by

$$T = \kappa/2\pi \tag{2.8}$$

where κ is the surface gravity, and given by

$$\kappa = \tfrac{1}{2}\,\partial C/\partial r \tag{2.9}$$

evaluated at the relevant root r_1 or r_2.

If $\kappa_1 \neq \kappa_2$, then there will be a temperature difference between the two horizons. Energy will then flow from one to the other, as a result of which M and the horizon areas will change. Assuming no charged particle flow, Q remains fixed. So does Λ. Differentiating (2.6) and (2.7) and using the definition of κ for each root of interest, one obtains

$$dS \equiv \tfrac{1}{4}\,dA = (1/T_1 - 1/T_2)\,dM \tag{2.10}$$

It follows that $dS = 0$ only if $T_1 = T_2$, otherwise $dS > 0$, on the assumption that mass-energy will flow from hot to cold.

This argument is independent of the detailed from of $C(r)$. One could, for example, arrive at the same conclusion even for the case that matter is present between the horizons, e.g., a static shell surrounding the black hole (Davies *et al.*, 1986). By definition, the coefficient of the $1/r$ term in $C(r)$ is the mass M. Putting $C = C(r, M/r)$, we have, at the horizons,

$$dC = (1/r)\,dM + (\partial C/\partial r)\,dr = 0 \tag{2.11}$$

From the definition of surface gravity, (2.9), we may rewrite (2.11) as

$$dM = \tfrac{1}{2}\kappa r\,dr = T\,dS$$

whence, in the case of two horizons, (2.10) follows.

The foregoing shows quite generally that attributing entropy to the cosmological horizon using (1.1) is consistent with a generalized second law. It might be objected, however, that this demonstration neglects the entropy of the radiation that fills the gap between the horizons. This entropy will change both because the horizon temperatures change as a result of the energy flow, and because the total volume between the horizons changes. But here we encounter one of the difficulties mentioned above [point (d)]. The entropy of de Sitter space is that of the horizon only: there *is* no actual de Sitter radiation to possess an entropy. On the other hand, in asymptotically flat spaces the black hole radiation *does* possess entropy, which must be added to that of the black hole horizon entropy. But when the black hole is in de Sitter space, it is not obvious how one should quantify the radiation entropy. How can one say whether a given quantity of radiation "belongs" to the black hole or the cosmological horizon?

These difficulties may be circumvented by considering certain definite initial and final states, where the radiation entropy cannot decrease, and comparing total horizon areas. As a first example, consider the black hole to be confined within a massless membrane that traps radiation. The black hole will come into stable equilibrium with the radiation within the membrane. If the volume enclosed by the membrane is small, we may neglect the entropy of the trapped radiation compared to that of the sum of the horizons. There will (for reasons discussed above) be no entropy associated with any de Sitter radiation outside the membrane.

Now suppose that the membrane is removed, allowing the radiation to escape and the black hole to evaporate. Eventually all the radiation produced by the hole will pass across the cosmological horizon, and the system will settle down to de Sitter space, with zero radiation entropy once again. For the quartic (2.6) the sum of all four roots vanishes, while the sum of the squares is $2/\Lambda$. These two conditions taken together imply

$$r_1^2 + r_2^2 + r_1 r_2 < 1/\Lambda \tag{2.12}$$

hence

$$A = 4\pi(r_1^2 + r_2^2) < 4\pi/\Lambda \tag{2.13}$$

But the final de Sitter horizon area is $4\pi/\Lambda$, so (2.13) implies that the final horizon area exceeds the initial area, in conformity with the generalized second law.

A black hole with nonzero Q will evaporate completely only so long as it can divest itself of the charge, by the radiation of charged particles. For this purpose it must be hot enough, as there are no massless charged particles. The temperature must therefore exceed the rest mass of the electron, otherwise charged particle production will be suppressed by the

Boltzmann factor $\exp(-mc^2/kT)$. For a black hole in asymptotically flat spacetime the nature of the evaporation depends (Davies, 1977) on the ratio Q^2/M^2. If, initially, $Q^2 < 0.86M^2$, then the specific heat of the hole is negative, and the hole gets hotter as it radiates. If, however, the hole still does not get hot enough to radiate charged particles, the mass will eventually fall below the critical value $M^2 = Q^2/0.86$ and the specific heat will become positive. After this the temperature *falls* as energy is radiated, further suppressing any charged particle production. The evaporation will then asymptote at $M^2 = Q^2$, at which the temperature of the hole is zero.

The case of a black hole in de Sitter space is similar, although the various parameters will be corrected by Λ terms. For example, there is the curious feature that zero temperature now occurs at a value $M^2 < Q^2$: the charge *exceeds* the mass. (This condition corresponds to a naked singularity in the $\Lambda = 0$ case.) In the chargeless case, the black hole is always hotter than the cosmological horizon (except when the roots $r_1 = r_2$ and the two horizons coincide). Therefore, the hole always evaporates completely. For nonzero Q one has the curious possibility that, if M^2/Q^2 is small enough, the temperature of the hole can be *less* than that of the cosmological horizon. It will then happen that energy will flow from the cosmological horizon into the hole until (stable) equilibrium is achieved.

One can therefore envisage the following scenario. Initially the black hole has zero temperature. There will thus be zero radiation entropy in the space between the horizons. The two horizons are now allowed to come into equilibrium at some $T > 0$. If the total horizon area at equilibrium is greater than the initial area, then the second law is satisfied, because any emitted radiation that accumulates between the horizons only contributes positively to the total entropy.

To study this scenario as a further test of the second law, I first investigate the condition for the black hole to have a common temperature with the cosmological horizon. A solution is (Mellor and Moss, 1989)

$$\begin{aligned} M^2 &= Q^2 \\ \kappa_1 = \kappa_2 &= [\Lambda(1 - 4m\Lambda^{1/2})]^{1/2} \\ r_1 &= \tfrac{1}{2}\Lambda^{-1/2}[1 + (1 - 4M\Lambda^{1/2})^{1/2}] \\ r_2 &= \tfrac{1}{2}\Lambda^{-1/2}[1 - (1 - 4M\Lambda^{1/2})^{1/2}] \end{aligned} \tag{2.14}$$

from which we obtain

$$A = 4\pi(r_1^2 + r_2^2) = 4\pi(1 - 2|Q|\Lambda)^{1/2}/\Lambda \tag{2.15}$$

Now consider the case of the black hole at zero temperature. The condition $\partial C/\partial r = 0$ at $r_2 = 0$ is equivalent to the condition for equal roots $r_2 = r_3$, where r_3 is the smallest positive root of (2.6). (The remaining root, r_4, is always negative.) It is also the condition for the limiting value of Q^2/M^2 before the horizon disappears and the black hole turns into a naked singularity. In this limiting case, (2.6) may be factorized as

$$(r-r_1)(r-r_2)^2(r-r_4)=0 \tag{2.16}$$

Comparison of coefficients of powers of r in (2.6) and (2.16) gives four equations for the three roots in terms of M, Q, and Λ. Eliminating r_1, r_4, and Λ yields a quadratic equation for r_2. The relevant root of this equation is

$$r_2=3M/2-(9M^2/4-2Q^2)^{1/2} \tag{2.17}$$

The four equations also yield for the total event horizon area

$$A=4\pi(r_1^2+r_2^2)=4\pi[1-2(Mr_2/\Lambda)^{1/2}]/\Lambda \tag{2.18}$$

A further complicated equation gives a relationship between M, Q, and Λ.

This initial state will be transformed into the final state by the flow of thermal mass-energy from the cosmological horizon into the black hole. As a result, M will rise. However, both Λ and Q remain fixed. As the final state has $M^2=Q^2$, it follows that, for the initial state, $M^2<Q^2$. A comparison of (2.15) and (2.18) reveals that, given this condition, the event horizon area of the final state exceeds that of the initial state, in conformity with the generalized second law.

3. MORE GENERAL HORIZONS

The success of the generalized second law of thermodynamics for black hole and de Sitter horizons prompts the question of whether attributing an entropy $\frac{1}{4}A$ to the event horizon area is valid under all circumstances. Many cosmological models contain event horizons, but they are not static. Moreover, particle detectors generally do not have a thermal response in these models. The thermodynamic connection of the horizon is therefore less obvious.

The first step in investigating this topic is to establish an area theorem for cosmological horizons analogous to Hawking's area theorem for black holes. For Friedmann-Robertson-Walker models with scale factor $a(t)$, uniformly filled with matter with energy density ρ and pressure p, the following theorem may be proved (Davies, in press-b).

Theorem. If $\rho+p\geq 0$ and $a(t)\to 0$ as $t\to\infty$, then the event horizon area is a nondecreasing function of time.

The theorem is valid for all three values of the curvature parameter $k=0$, ± 1. The condition $\rho+p\geq 0$ is known as the dominant energy condition, and is identical to that needed to prove the black hole area theorem.

Again, proceeding by analogy with black holes, one is prompted to consider what happens if the dominant energy condition is relaxed. In the black hole case this comes about because the quantum vacuum expectation value of the energy density near the hole is negative, allowing a flux of negative energy to stream into the hole. As a result, the mass, and hence horizon area, of the hole decreases. The loss in entropy that this represents is offset by the entropy of the thermal Hawking radiation which is emitted into the hole's environment. In this way the generalized second law of thermodynamics remains valid.

In the cosmological case, a natural way to relax the dominant energy condition is to introduce a bulk viscosity into the cosmological fluid. If the fluid has equation of state $p=(\gamma-1)\rho$ and bulk viscosity $\eta=\alpha\rho$ ($\alpha=$ const >0), then the effective pressure is given, not by p, but by $p'=p-3H\alpha\rho$. If one chooses $\gamma<3H\alpha$, then $p'<-\rho$, thus violating the energy condition.

The Friedmann equations can be solved exactly (Barrow, to appear) in the $k=0$ case to give

$$\ln a + Ca^{3\gamma/2} = \gamma(t-t_0)/3\alpha \tag{3.1}$$

where C and t_0 are constants. The radius of the event horizon is defined as

$$R(t)=a(t)\int dt'/a(t') \tag{3.2}$$

where the integral is taken over the whole future from the time t of interest. This quantity can also be evaluated exactly for this model.

If $\alpha=\gamma=0$, one recovers the normal de Sitter solution (2.1). For the purposes of investigating the second law, I consider only small departures from this solution, with $\gamma\approx 0$ and $\alpha \ll 1/H$. Then from the exact solution one obtains (Davies, in press-b)

$$R\approx 1/H \tag{3.3}$$

$$\dot{R}\approx -9H\alpha/2 \tag{3.4}$$

whence

$$\tfrac{1}{4}\dot{A}=2\pi R\dot{R}\approx -9\pi\alpha \tag{3.5}$$

We see that the effect of viscosity in this case is to cause the horizon area to decrease with time. However, the very viscosity which produces this decrease also generates heat, and hence entropy, in the cosmological material. The question then arises as to whether the latter will offset the former.

Although a particle detector will no longer register a precisely thermal spectrum, if the departure from de Sitter expansion is small, then one can define an "instantaneous temperature" by supposing that the detector is switched on and off slowly, yet still on a short time scale compared to $H/\dot{H}$. It can then be shown that the detector registers a slowly varying Hawking-type temperature

$$T = 1/2\pi R \tag{3.6}$$

We take this to be the effective irreducible temperature of the cosmological fluid. The rate of entropy production within the horizon volume $V = 4\pi R^3/3$ due to the viscosity is given by (Davies, in press-b)

$$\dot{S}_m = 9H^2\alpha\rho V/T \tag{3.7}$$

The Friedmann equations give

$$\rho = 3H^2/8\pi \tag{3.8}$$

so, using (3.3), (3.6), and (3.8) in (3.7), we find

$$\dot{S}_m \approx 9\pi\alpha \tag{3.9}$$

which is the negative of (3.5). It follows that the generalized second law of thermodynamics remains valid even in this case, if one assigns a gravitational entropy to the cosmological event horizon area, using the relationship (1.1), as for the black hole.

4. DISSIPATIVE STRUCTURE

It is interesting to note that (3.1) possesses a special solution:

$$H = \gamma/3\alpha = \text{const} \tag{4.1}$$

This corresponds to de Sitter space, but it is not the usual de Sitter solution (except in the limit $\alpha = \gamma = 0$), which is devoid of matter. Here, there is cosmological material which generates entropy through viscosity, and achieves a dynamic equilibrium by exporting this entropy across the event horizon at a constant rate. This entropy and energy flow is maintained by a temperature differential between the fluid and the horizon. The contents of the universe are therefore not in *thermodynamic* equilibrium with the horizon, although the system is in *gravitational* equilibrium. The system resembles a dissipative structure of nonequilibrium thermodynamics (e.g., Nicolis and Prigogine, 1977). It is also a curious form of the old steady-state universe of Hoyle (1948). However, it should be pointed out that this particular cosmological dissipative structure is unstable. It evolves away from the steady-state form toward power-law Friedmann-type behavior.

Barrow (to appear) has discussed other viscous-driven cosmological models in which de Sitter space is a *stable* attractor as $t \to \infty$.

To maintain the steady-state condition, T, p, ρ, and the entropy density s of the cosmological fluid must all remain constant with time. Considering the total entropy S in a comoving volume a^3 and applying the first law of thermodynamics, we find

$$sT = \rho + p = \gamma\rho \tag{4.2}$$

Then, using (2.3) and (3.8), we obtain

$$sT = (3\pi\gamma/2)T_h^2 \tag{4.3}$$

where T_h is the horizon temperature. The entropy density of the fluid cannot exceed that of thermal radiation at temperature T, so

$$sT \leq 4aT\mathrm{a}/3 \tag{4.4}$$

where a is the radiation constant, equal to $\pi^2/60$ in our units. From (4.3) and (4.4) one obtains

$$T \gtrsim (135\gamma/2\pi)^{1/4}(T_h T_{Pl})^{1/2} \tag{4.5}$$

where T_{Pl} is the Planck temperature. For consistency we also require $T_{Pl} \gg T \geq T_h$.

It is interesting to note that from the point of view of the gravitational field, de Sitter space is time symmetric. However, the cosmological material possesses an arrow of time defined by the direction of entropy production and flow. This comes in turn from the arrow of time defined by the viscosity, which is an irreversible process. The time directionality of the viscosity depends upon an assumption about the microscopic state of the fluid, such as the absence of correlations between particles in the past (but not the future). At the present level of theory one simply assumes an equation of state to maintain steady-state conditions, and this gives no information about the microscopic state of the fluid. It merely requires that the mass-energy density remains constant as the universe expands. A suitable microscopic theory would have to account for the continuous creation of new particles to maintain this requirement. Such a microscopic theory, which might be along the lines of the so-called creation field (or C field) of Hoyle and Narlikar (1974), would need a quantum mechanical description. The spontaneous uncorrelated creation (rather than annihilation) of particles in the expanding universe would then define an arrow of time. This is usually implicit in the various treatments of particle creation in expanding universes (e.g., Birrell and Davies, 1981, Chapter 8), and its origin can be traced to an assumption about the nature of the initial quantum state, such as random phases.

5. GRAVITATIONAL ENTROPY

The theorem mentioned in Section 3 is valid only so long as $a \to \infty$ as $t \to \infty$. This is not the case for a universe that reaches a maximum value of a and then recontracts to a final singularity. In that case the integral in the expression (3.2) for the radius of the event horizon (and the corresponding modifications for the $k = \pm 1$ models) must be truncated at the final singularity, $t = t_f$. As $t \to t_f$, so $R_\mathrm{h} \to 0$ (hence $A \to 0$) in all cases. As we have seen, it can still be the case that a *generalized* second law remains valid, even as the horizon shrinks, due to viscous generation of entropy in the cosmological fluid. However, one could still consider those cosmological models which contract to a final singularity without viscosity, e.g., the $k = +1$ radiation-filled Friedmann model with zero cosmological constant. In this particular model, for example, $a \propto (t_f - t)^{1/2}$ and $R_\mathrm{h} \propto t_f - t$ as $t \to t_f$. The radiation entropy in a comoving volume a^3 is constant, so the entropy density is proportional to $a^{-3} \propto (t_f - t)^{-3/2}$. Hence the total radiation entropy in a horizon volume is proportional to $(t_f - t)^{3/2}$, which also tends to zero as $t \to t_f$. It follows that both the radiation entropy and the horizon entropy decrease monotonically as the final singularity is approached.

The existence of such models signals a clear violation of the generalized second law of thermodynamics. One might take three different positions concerning this result. The first is to accept it at face value. We are used to regarding the second law of thermodynamics as an absolutely inviolable law of nature, but why must this be so? Arguments in defense of the second law emphasize the paradoxical consequences that could attend a violation, such as the construction of a *perpetuum mobile*. It may be the case, however, that in this broader generalized form, a violation of the second law will not automatically lead to such paradoxical consequences. For example, one might consider a Carnot cycle to transfer heat from a cold to a hot body without the expenditure of work, using in some way the thermodynamic properties of the event horizon in a contracting universe [following the idea of "mining" the de Sitter horizon by a box-on-a-rope mechanism (Davies, 1984*b*)]. It might then be found that such a cycle could not be completed in the time available before the universe reaches an end at the final singularity. Only detailed calculations can settle this point.

The second position is to question the assumption that the gravitational entropy is always given by (one-quarter of) the event horizon area. One might conjecture that some more elaborate dependence on the spacetime structure is called for. (Indeed, such a conjecture has been made by Penrose and others—see below.) For example, the entropy might be a functional of geometry that reduces to event horizon area in static spacetimes, but has a more complicated form when the horizon area is changing with time. Possibly a simple sum of horizon area and some other functional of geometry (which reduces to an additive constant in the static case) would work. To rescue the second law for a recontracting Friedmann model, the overlooked portion of the gravitational entropy would need to rise sufficiently fast (but could still remain finite) as the universe contracts to the final singularity.

The third point of view is to regard the recontracting Friedmann model as unstable, i.e., the initial data form a set of measure zero in the space of initial conditions. It is known that, generically, recollapsing universes are unstable against the growth of anisotropy. This might in itself rescue the second law, if the anisotropization of the horizon causes the area to grow faster than the contraction causes it so shrink. However, I wish to consider here a more interesting possibility.

It has long been appreciated that a self-gravitating system defines a gravitational arrow of time because, roughly speaking, it grows more clumpy with time. The general trend to go from "smooth to clumpy" should, one supposes, be expressible in terms of a growth of gravitational entropy. This suggests that gravitational entropy should be defined in terms of some measure of the "degree of clumpiness" of the gravitational field. Penrose (1979), for example, has suggested that the Weyl conformal tensor might provide such a measure. Here I wish to propose that the relevant quality of the field is not its departure from conformal flatness, but its *complexity*. A clumpy gravitational arrangement is certainly more complex than a smooth one.

Assuming that complexity really does capture the essential irreversible quality of gravitational time development, it is obviously necessary to have some precise way of quantifying it. Recently algorithmic complexity theory has been proposed as a formalism for quantifying complexity. In particular, Zurek (to appear) has defined an "algorithmic entropy" based on the work of Bennett (1982). Zurek's definition has the important virtue that it is not formulated in terms of ensembles, but is meaningful for a single state of the system. This is in contrast to the usual definition, based on Shannon's information theory, where the entropy is taken as a measure of our ignorance of the system, i.e., in terms of missing information.

Roughly speaking, the algorithmic complexity of a state is the number of bits of information contained in the minimal computer algorithm that can simulate the system. A more complex system obviously requires a more elaborate algorithm. According to Zurek, the physical entropy is the sum of the missing information (i.e., Shannon entropy) and the length of the most concise record expressing the information about the state already at hand. For thermodynamic equilibrium this definition reproduces the usual results of statistical mechanics. Consider, for example, a gas in thermodynamic equilibrium. The molecules are distributed at random. With the usual coarse graining one would assign a certain entropy to the gas. If, however, we could inspect the gas more closely, the complexity of the chaotic arrangement of molecules would be seen to be greater. Thus, the length of the record goes up. On the other hand, because of the closer scrutiny of the gas, the missing information would have gone down. These two changes compensate.

It is interesting to apply these ideas to the gravitational case. First consider a black hole. It is in the nature of the event horizon that observers outside the hole have no access to information about the interior state, except for the mass, certain charges, and the angular momentum. As pointed out by Hawking, this gives black hole entropy an "objective" quality. That is, we cannot choose to inspect the interior of the hole more closely. The black hole uniqueness theorems ensure that the hole is algorithmically very simple to describe. Therefore the physical entropy of the hole consists almost entirely of missing information.

Now consider the case of the growth of clumpiness of a gravitational field, without an event horizon. An initially smooth field requires a relatively short algorithm to specify it. By contrast, a clumpy field is very complex and requires a much longer algorithm. In general, there will also be some missing information due to our imperfect knowledge of the field. However, in principle we could inspect the field with arbitrary resolution, in which case the algorithmic record would be longer, and the information entropy could be reduced to zero.

I conjecture that in a recontracting universe the growth in anisotropy and inhomogeneity that is generically expected represents a rise in complexity and hence in algorithmic entropy that more than compensates for the combined loss of matter entropy and horizon area. I hope to present a detailed study of this conjecture elsewhere.

6. COSMIC ENTROPY CENSOR

It is often the case that an accepted theory can admit solutions which are unpalatable or even paradoxical on physical grounds. Under those circumstances, rather than reject the theory, one may prefer to append to it a statement forbidding such unacceptable solutions. The most obvious examples of this concern questions of causality. Special relativity admits the existence of tachyons, but because they would enable signals to be sent into the past, they are usually ruled out. Similarly, in general relativity, there are solutions to Einstein's field equations with closed timelike world lines, which are equally problematical from the point of view of causality. Singularities in general relativity likewise pose a problem, because they represent a breakdown of causality. Especially unpleasant are "naked" singularities, which form from nonsingular initial conditions, and represent a breakdown of causality at some finite epoch in the history of the universe. Usually the so-called cosmic censorship hypothesis of Penrose (1969) is invoked to rule out such a situation.

Potential violations of the second law of thermodynamics are really in the same category. After all, the second law imposes on the universe an arrow of time, which defines the directionality of the cause $\rightarrow$ effect relationship. A violation of the second law could lead to an inversion of that relationship, letting "time run backward" and opening the way to the same sort of paradoxical effects as with tachyons and closed timelike world lines, i.e., enabling signals to be sent into the past. It seems reasonable to invoke an "entropy censor" to forbid such circumstances.

This conjecture becomes especially interesting if it is extended to the generalized second law of thermodynamics, because it then makes a strong statement about spacetime structure. A weak form of entropy censorship hypothesis is to rule out any solution of the gravitational field equations in which the sum of the gravitational and matter entropy decreases with time at any epoch. Such a hypothesis eliminates a very large class of spacetimes, including, for example, all so-called hyperinflationary models [where the scale factor increases faster than an exponential, e.g., as $\exp(t^2)$] in which viscosity fails to generate entropy fast enough to keep pace with the contracting horizon area. On the face of it, this weak hypothesis would also rule out a recontracting Friedmann model unless some additional contribution to the gravitational entropy is discovered.

One might also entertain a strong entropy censorship principle, in which any *theory* which admits violations of the weak principle is rejected. The justification for the strong hypothesis is that, given a universe described by such a theory, a sufficiently advanced intelligence could in principle manufacture a decrease of entropy. In the case of ordinary thermodynamics this does not pose a problem. It is well known that, in principle, a Maxwell demon could achieve a violation of the second law. However, in order to operate, the demon has to inspect the microscopic state of a gas, and this procedure itself generates entropy at a compensatory rate (Bennett, 1987).

When it comes to gravitational entropy, the situation is different. Horizon-decreasing solutions require only that appropriate global initial conditions are established and not, as with the Maxwell demon, that microscopic degrees of freedom are manipulated continuously. There would seem to be no restrictions of an informational or algorithmic nature on the establishment of horizon-decreasing initial conditions in those theories that admit such solutions. The strong entropy censorship hypothesis would be a very restrictive filter of both matter and gravitational field theories. Almost all Lagrangians would be inconsistent with the hypothesis.

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